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A Model of Prognosis of Immersion of a Pile into the Soil During its Compaction

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Abstract

In this paper we present a model of pile driving into soil during its compaction. Based on the model we analyzed the pile depth and the possibility of its control. We also presented an analytical approach for prognosis of the introduced model. The approach gives a possibility to take into account changes of parameters in space and time, as well as the nonlinearity of the considered proc-esses.

Keywords: Pile driving into soil, Process model, Analytical approach of analysis.

1 | Introduction

The volume of construction of high-rise buildings is constantly increasing. In their construction, as well as in the construction of other structures, pile foundations are used. In connection with the tendency to reduce the cost of construction in general, a special role is played by reducing the costs of erecting the underground part of buildings. Other tasks follow from this task - optimization of pile foundations and quantitative assessment of the effectiveness of this optimization. When planning pile foundations, it is necessary to analyze the immersion of the pile and soil compaction under multiple impacts. Currently, there are a large number of works on this analysis [1-7]. This paper presents a model of pile immersion in the soil during its compaction. Based on the model presented in this paper, the depth and speed of pile immersion are analyzed. We also presents an analytical approach of analysis of the considered model. The approach gives a possibility to take into account changes of parameters in space and time, as well as the nonlinearity of the considered processes.

2 | Method of Solution

The deepening of the pile into the ground under the action of the pile driver is determined using the following type of equation [8], [9]

$$m_p a = m_p g + \sum_{n=1}^{N} Q_n(t) - R(t), \tag{1}$$

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where $R(t) = g\left[m_p + m_{pd} + \frac{H(t)m_p^2}{S(t)(m_p+m_{pd})}\right]$.

Is the soil reaction force; mp is the pile mass; mpd is the pile driver mass; a is the pile acceleration; g is the gravity acceleration; $Q_n(t) = Q_{0n}\delta(t-\tau_n)$ is the force of the n-th hit of the pile driver; $\delta(t-\tau_n)$ is the Dirac delta function [8]; τ_n is the moment of impact of the pile driver on the pile; H(t) is the height of fall of the pile driver; S(t) is the depth of penetration of the pile into the ground under the impact of the pile driver; L-pile length. The following conditions are imposed on the depth of penetration of the pile into the ground under the impact of the pile driver: S(0)=0, S(Θ)=L. *Eq. (1)* can be represented in the following form

$$\frac{d^2H(t)}{dt^2} = g + \frac{1}{m_p}\sum_{n=1}^{N} Q_n(t) - g\left[1 + \frac{m_{pd}}{m_p} + \frac{H(t)m_p}{S(t)(m_p+m_{pd})}\right]. \quad (2)$$

We will find the solution of this equation using the appropriate method for solving ordinary differential equations [10]. To solve the equation we transform it to the following form

$$\frac{d^2H(t)}{dt^2} - \frac{H(t)m_p g}{S(t)(m_p+m_{pd})} = g + \frac{1}{m_p}\sum_{n=1}^{N} Q_n(t) - g\left(1 + \frac{m_{pd}}{m_p}\right). \quad (3)$$

Next we solve the equation in the framework two-step algorithm. At the first step of the procedure we remove the right side of the equation. In this situation we obtain the following equation

$$\frac{d^2H(t)}{dt^2} - \frac{H(t)m_p g}{S(t)(m_p+m_{pd})} = 0. \quad (4)$$

Now we will solve the above equation in the following form

$$+H(t) = \exp(\lambda t), \quad (5)$$

where λ is not yet known parameter. Substitution of the *Function (4) into Eq. (3)* leads to the following equation to determine values of the parameter λ

$$\lambda^2 = \frac{m_p g}{S(t)(m_p+m_{pd})}. \quad (6)$$

The equation has two solutions

$$\lambda_1 = \sqrt{\frac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}. \quad (7)$$

In this situation solution of *Eq. (7)* could be written as

$$H(t) = C_1 \exp\left[-t\sqrt{\frac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right] + C_2 \exp\left[t\sqrt{\frac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right], \quad (8)$$

where C_1 and C_2 are the integration constants. Next we assume, that C_1 and C_2 will be no yet known functions of time t, i.e. $C_1=C_1(t)$, $C_2=C_2(t)$. Now we will substitute *Relation (6)* with account time dependence of functions $C_1(t)$ and $C_2(t)$ into *Eq. (3)*. After that we obtain system of equations to determine functions $C_1(t)$ and $C_2(t)$ in the following form

$$\begin{cases} \dfrac{dC_1(t)}{dt}\exp\left[-t\sqrt{\dfrac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right]+\dfrac{dC_2(t)}{dt}\exp\left[t\sqrt{\dfrac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right]=0 \\ -\dfrac{dC_1(t)}{dt}\exp\left[-t\sqrt{\dfrac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right]+\dfrac{dC_2(t)}{dt}\exp\left[t\sqrt{\dfrac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right]= \\ =\left[g+\dfrac{1}{m_p}\sum_{n=1}^{N}Q_n(t)-g\left(1+\dfrac{m_{pd}}{m_p}\right)\right]\Big/ S(t)\sqrt{\dfrac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}. \end{cases} \tag{9}$$

Adding *Eq. (9)* leads to an equation for determining the derivative of the function $C_2(T)$ in the following form

$$\frac{dC_2(t)}{dt}\exp\left[t\sqrt{\frac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right]=\left[g+\frac{1}{m_p}\sum_{n=1}^{N}Q_n(t)-g\left(1+\frac{m_{pd}}{m_p}\right)\right] \times\frac{1}{2S(t)}\sqrt{\frac{(m_p+m_{pd})\int_0^t S(\theta)d\theta}{m_p g}}. \tag{10}$$

Transformation of the above relation leads to the following result

$$\frac{dC_1(t)}{dt}=\frac{1}{2S(t)}\left[g+\frac{1}{m_p}\sum_{n=1}^{N}Q_n(t)-g\left(1+\frac{m_{pd}}{m_p}\right)\right]\exp\left[t\sqrt{\frac{m_p g}{(m_p+m_{pd})\int_0^t S(\theta)d\theta}}\right] \times\frac{1}{2S(t)}\sqrt{\frac{(m_p+m_{pd})\int_0^t S(\theta)d\theta}{m_p g}}. \tag{11}$$

Integration of the *Relation* (9) on time leads to the following result gives a possibility to obtain function $C_2(t)$ in the final form

$$C_2(t)=\frac{1}{2}\int_0^t\left[g+\frac{1}{m_p}\sum_{n=1}^{N}Q_n(\tau)-g\left(1+\frac{m_{pd}}{m_p}\right)\right]\sqrt{\frac{(m_p+m_{pd})\int_0^\tau S(\theta)d\theta}{m_p g}}\exp\left[-t\sqrt{\frac{m_p g}{(m_p+m_{pd})\int_0^\tau S(\theta)d\theta}}\right] \times\frac{1}{2S(\tau)}\sqrt{\frac{(m_p+m_{pd})\int_0^\tau S(\theta)d\theta}{m_p g}}d\tau+\tilde{C}_2. \tag{12}$$

Subtraction of *Eq. (12)* leads to an equation for determining the derivative of the Function C1(T) in the following form

$$\frac{dC_1(t)}{dt}\exp\left[-t\sqrt{\frac{m_p g}{\left(m_p+m_{pd}\right)\int_0^t S(\theta)d\theta}}\right]=\frac{1}{2S(t)}\left[g+\frac{1}{m_p}\sum_{n=1}^{N}Q_n(t)-g\left(1+\frac{m_{pd}}{m_p}\right)\right] \times\sqrt{\frac{\left(m_p+m_{pd}\right)\int_0^t S(\theta)d\theta}{m_p g}}. \quad (13)$$

Transformation of the above relation leads to the following result

$$\frac{dC_1(t)}{dt}=\frac{1}{2S(t)}\left[g+\frac{1}{m_p}\sum_{n=1}^{N}Q_n(t)-g\left(1+\frac{m_{pd}}{m_p}\right)\right]\exp\left[t\sqrt{\frac{m_p g}{\left(m_p+m_{pd}\right)\int_0^t S(\theta)d\theta}}\right] \times\sqrt{\frac{\left(m_p+m_{pd}\right)\int_0^t S(\theta)d\theta}{m_p g}}. \quad (14)$$

Integration of the *Relation (12)* on time leads to the following result gives a possibility to obtain Function $C_1(t)$ in the final form

$$C_1(t)=\frac{1}{2}\int_0^t\left[g+\frac{1}{m_p}\sum_{n=1}^{N}Q_n(\tau)-g\left(1+\frac{m_{pd}}{m_p}\right)\right]\sqrt{\frac{\left(m_p+m_{pd}\right)\int_0^\tau S(\theta)d\theta}{m_p g}}\exp\left[t\sqrt{\frac{m_p g}{\left(m_p+m_{pd}\right)\int_0^\tau S(\theta)d\theta}}\right] \times\frac{1}{2S(\tau)}\sqrt{\frac{\left(m_p+m_{pd}\right)\int_0^\tau S(\theta)d\theta}{m_p g}}d\tau+\tilde{C}_1. \quad (15)$$

Substitution of *Functions (10)* and *(13)* into the *Function (6)* leads to the following result

$$H(t)=\left\{\int_0^t\frac{S(\tau)}{2m_p}\frac{\sum_{n=1}^{N}Q_n(\tau)-g\left(m_p+m_{pd}\right)}{\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)d\theta}}\exp\left[-2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)d\theta}\right]d\tau+C_1\right\} \times\exp\left[2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^t S^{-1}(\tau)d\tau}\right]+\exp\left[-2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^t S^{-1}(\tau)d\tau}\right]$$

$$\times\left\{\int_0^t \frac{S(\tau)}{2m_p}\frac{\sum_{n=1}^{N}Q_n(\tau)-g(m_p+m_{pd})}{\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)\,d\theta}}\exp\left[2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)\,d\theta}\right]d\tau+C_2\right\}.$$

where,

$$C_1=\left(\left\{L-\int_0^\Theta \frac{S(\tau)}{2m_p}\frac{\sum_{n=1}^{N}Q_n(\tau)-g(m_p+m_{pd})}{\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)\,d\theta}}\exp\left[-2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)\,d\theta}\right]d\tau\right\}\right.$$

$$\times\exp\left[2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\Theta S^{-1}(\tau)\,d\tau}\right]-\exp\left[-2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\Theta S^{-1}(\tau)\,d\tau}\right]$$

$$\times\left\{L-\int_0^\Theta \frac{S(\tau)}{2m_p}\frac{\sum_{n=1}^{N}Q_n(\tau)-g(m_p+m_{pd})}{\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)\,d\theta}}\exp\left[2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\tau S^{-1}(\theta)\,d\theta}\right]d\tau\right\}$$

$$\left.\times\left\{\exp\left[2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\Theta S^{-1}(\tau)\,d\tau}\right]-\exp\left[-2\sqrt{\frac{g\,m_p}{m_p+m_{pd}}\int_0^\Theta S^{-1}(\tau)\,d\tau}\right]\right\}\right).$$

3 | Discussion

This section analyzes the change in pile height above the ground over time during the process of driving it. *Figs. 1* show typical dependences of the considered height for different values of the pile driver impact force on time. Increasing of this force corresponds to decreasing of the considered height. *Fig. 2* shows typical dependences of the considered height for different values of the pile driver impact frequency on time. Increasing of this fo corresponds to decreasing of the considered height. *Fig. 3* shows typical dependences of the considered height for different values of the pile driver mass on time. Increasing of this of corresponds to decreasing of the considered height. *Fig. 4* shows typical dependences of the considered height for different values of the pile mass on time. Increasing of this of corresponds to decreasing of the considered height.

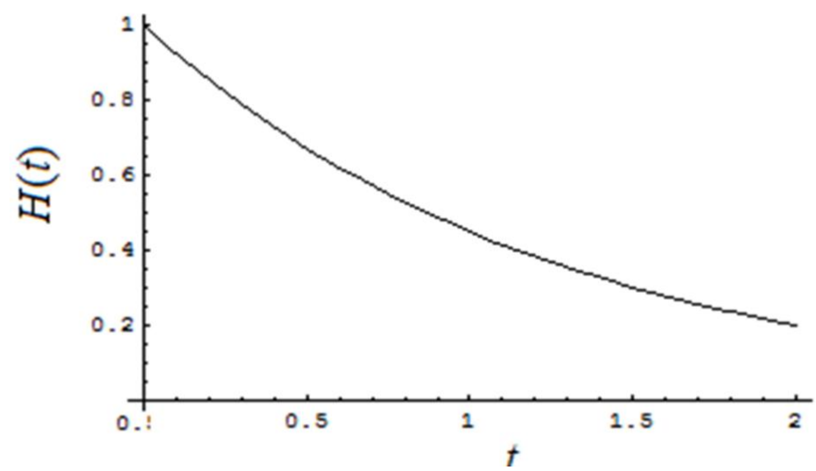

Fig. 1. Typical dependences of the considered height for different values of the pile driver impact force on time.

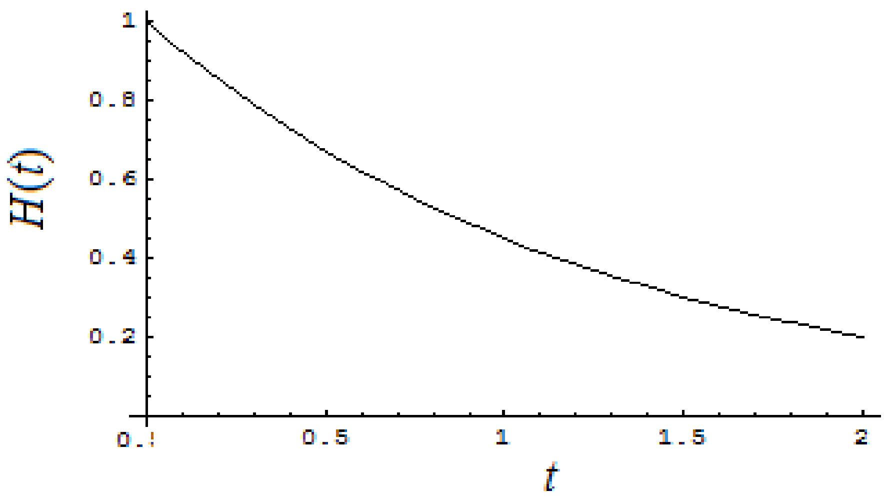

Fig. 2. Typical dependences of the considered height for different values of the pile driver impact force on time.

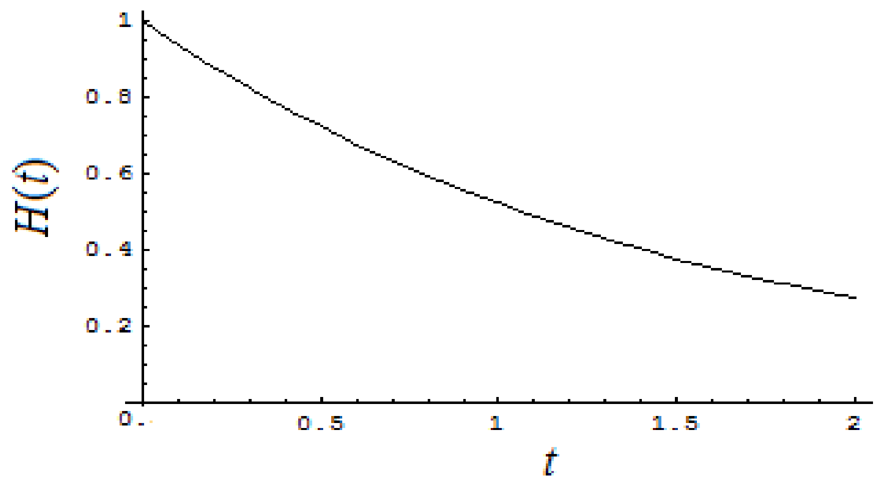

Fig. 3. Typical dependences of the considered height for different values of the pile driver impact force on time.

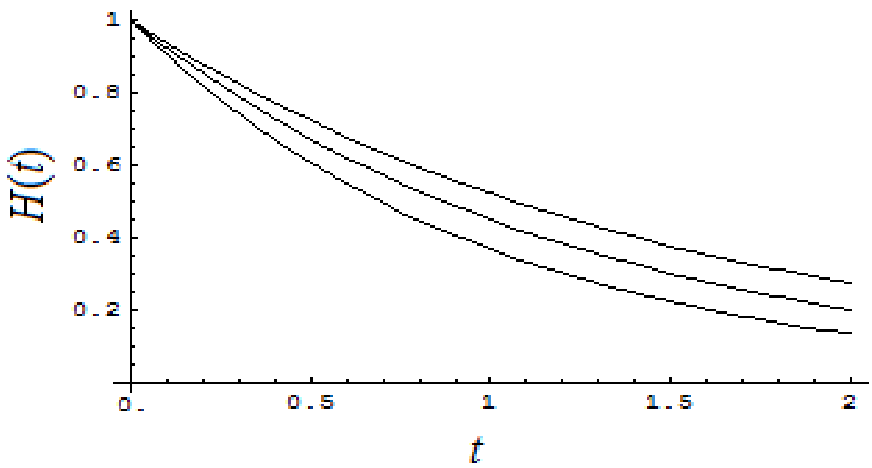

Fig. 4. Comparison of typical dependences of the considered height for different values of the pile driver impact force on time.

Increasing of this force corresponds to decreasing of the considered height.

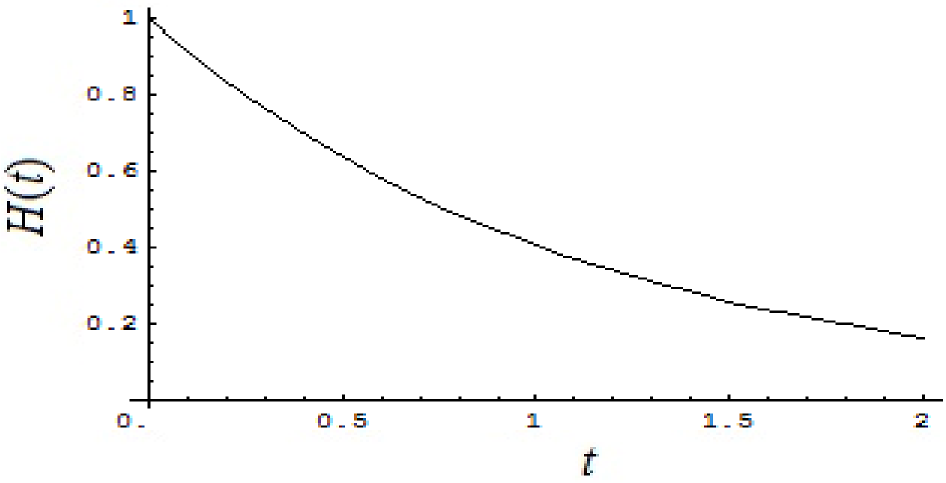

Fig. 5. Typical dependence of the considered height for different values of the pile driver impact frequency on time.

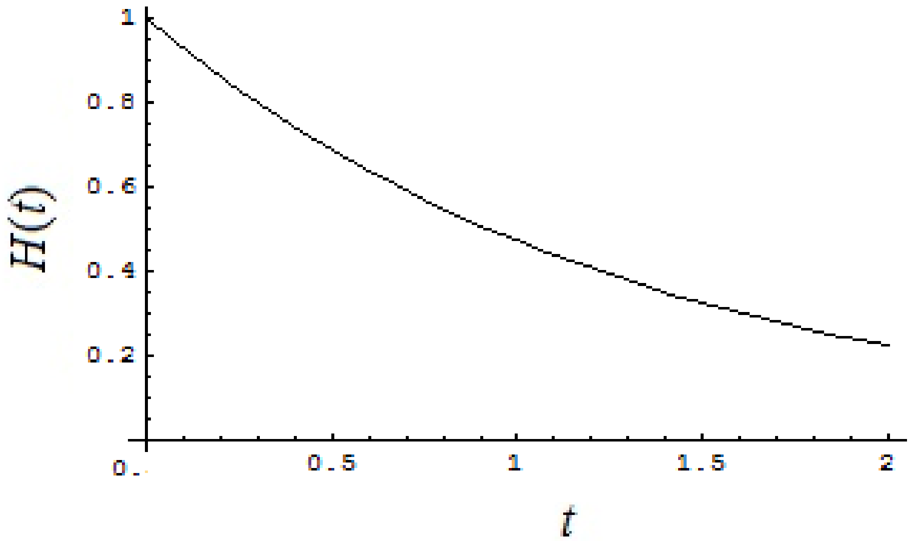

Fig. 6. Typical dependence of the considered height for different values of the pile driver impact frequency on time.

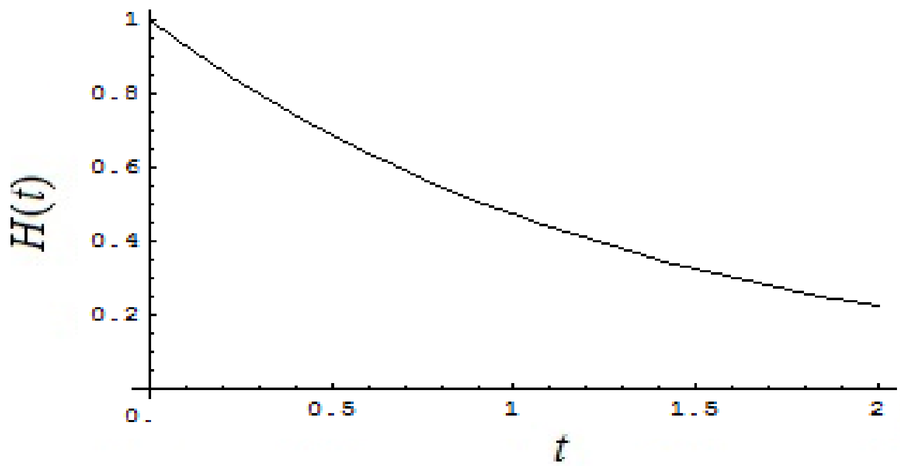

Fig. 7. Typical dependence of the considered height for different values of the pile driver impact frequency on time.

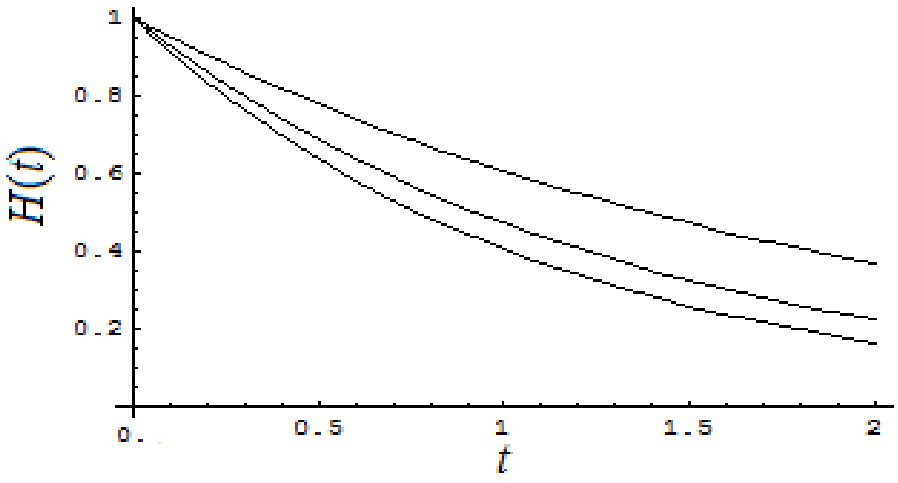

Fig. 8. Comparison of typical dependences of the considered height for different values of the pile driver impact frequency on time.

Increasing of this frequency corresponds to decreasing of the considered height.

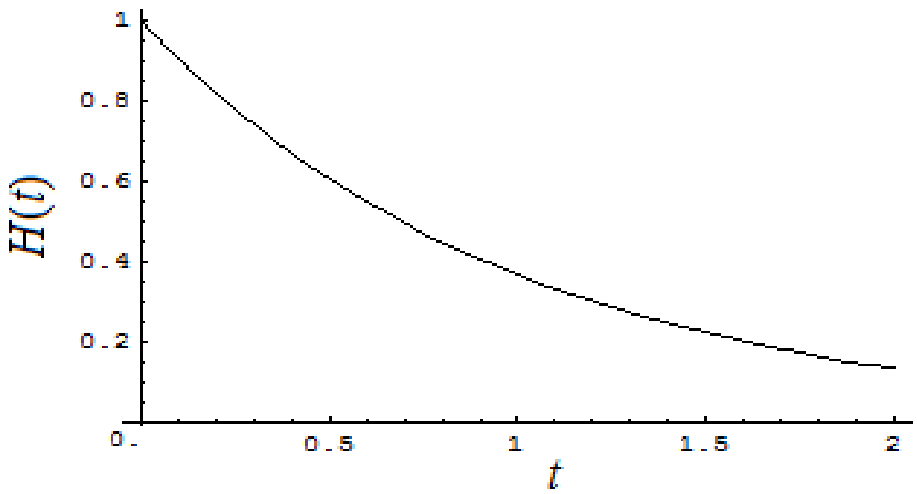

Fig. 9. Typical dependences of the considered height for different values of the pile driver mass on time.

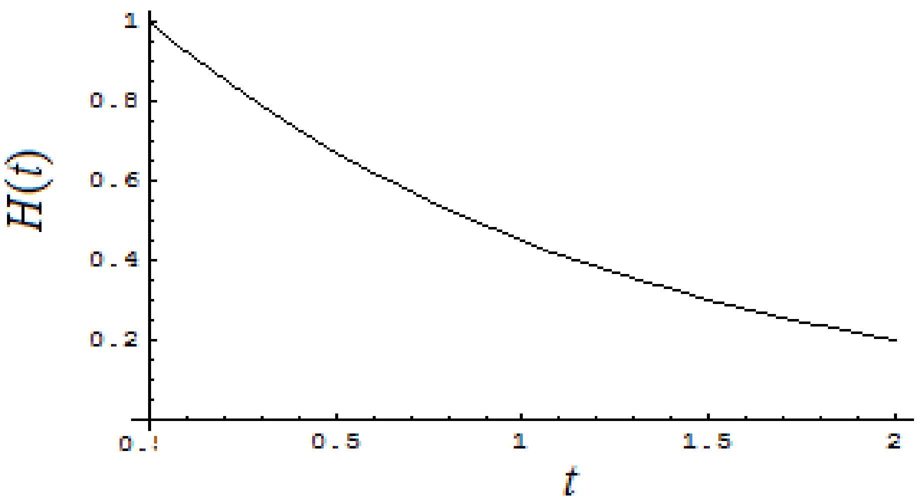

Fig. 10. Typical dependences of the considered height for different values of the pile driver mass on time.

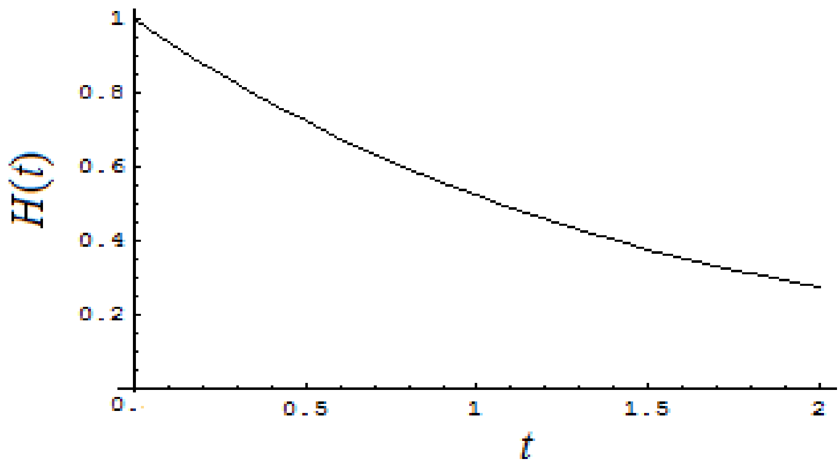

Fig. 11. Typical dependences of the considered height for different values of the pile driver mass on time.

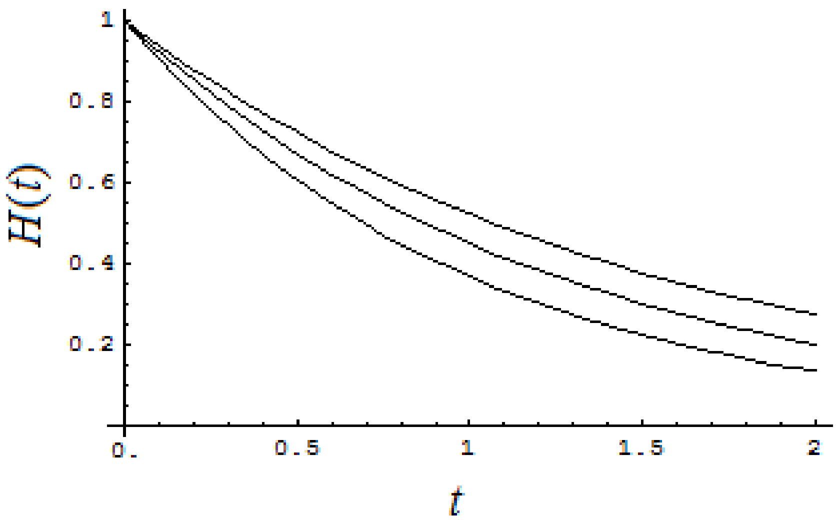

Fig. 12. Comparison of typical dependences of the considered height for different values of the pile driver mass on time.

Increasing of this force corresponds to decreasing of the considered height.

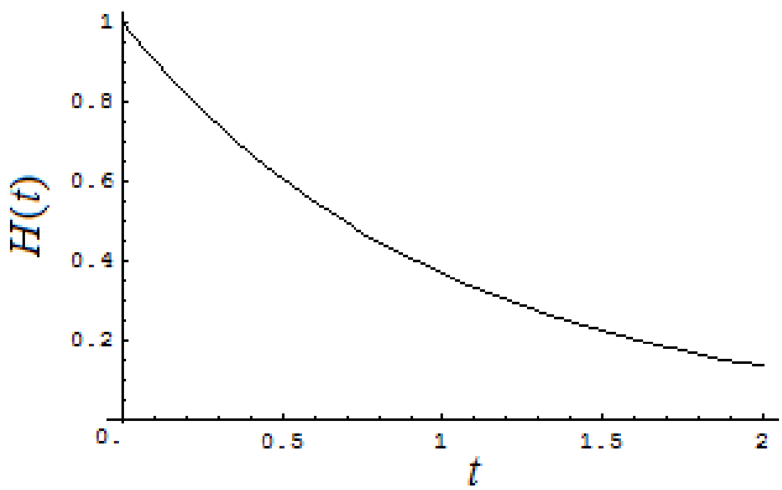

Fig. 13. Typical dependences of the considered height for different values of the pile mass on time.

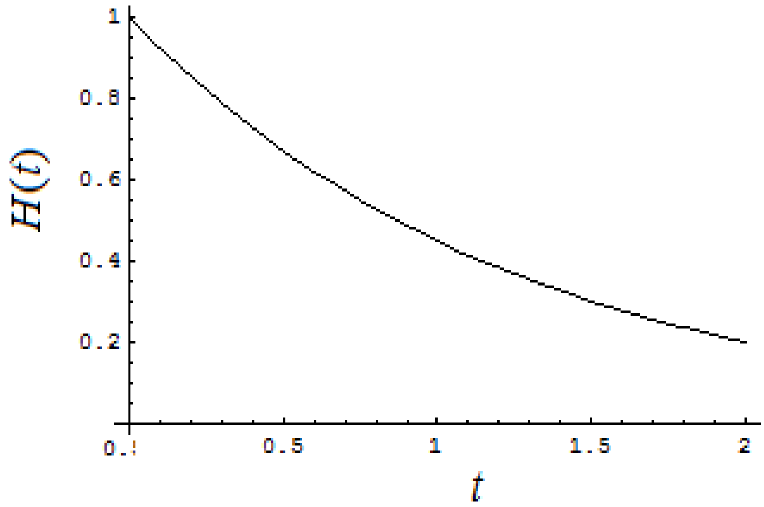

Fig. 14. Typical dependences of the considered height for different values of the pile mass on time

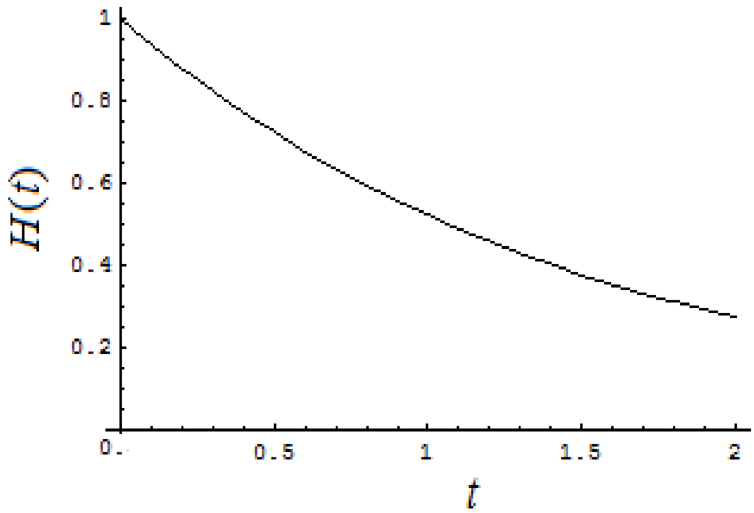

Fig. 15. Typical dependences of the considered height for different values of the pile mass on time.

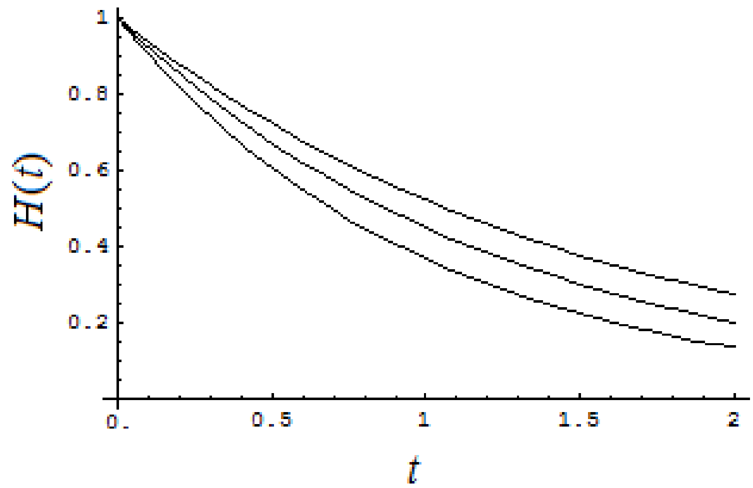

Fig. 16. Comparison of typical dependences of the considered height for different values of the pile mass on time.

Increasing of this force corresponds to decreasing of the considered height.

4 | Conclusion

We present a model of pile driving into soil during its compaction. Based on the model presented in this paper, we analyzed the pile depth and the possibility of its control. We also presented an analytical approach for prognosis of the introduced model. The approach gives a possibility to take into account changes of parameters in space and time, as well as the nonlinearity of the considered processes.

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Data Availability

All appropriate data are available.

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