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## On Analysis of Oscillations of a Multilayer Structures

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### Abstract

In this paper we introduce an approach to increase sound-insulation by using a multilayer building construction. As an example of such analysis we consider sound processing of multilayer enclosing and load-bearing constructions to increase sound-insulating. We also introduce an analytical approach for analysis of the considered processes.

The analytical results demonstrate that multilayer building structures significantly modify the spatio-temporal distribution of displacements under acoustic excitation. The proposed analytical approach, validated through comparison with direct numerical simulations, effectively captures the oscillatory behavior of the structure. The findings confirm that appropriately designed multilayer configurations can enhance sound-insulation performance while maintaining structural efficiency.


**Keywords:** Sound-insulation of building constructions, Analytical approach for prognosis, Structural engineering, Acoustic oscillation, Sound insulation, Wave equation, Civil engineering.

## 1 | Introduction

High rates of development of construction equipment create the necessary prerequisites for the design and construction of buildings and other structures of elements that have significant strength and stability with low weight and small thickness [1–8]. At the same time, the development of technology leads to the emergence of more powerful machines and to increasing number of vehicles, which leads to increasing of noise in populated areas, civil and industrial buildings. Acoustic improvement of premises becomes an actual problem of each design and construction of each building [9–13].

In the framework of solving of this problem, the problem of the sound- insulating ability of the enclosing and supporting structures is first of all solved. To solve the problem, it is necessary to analyze the sound effect on the structure. In the framework of this paper we analyzed sound- insulating enclosing and load-bearing building constructions. We will analyze this effect using the example of transverse oscillations of a multilayer construction under influence of a plane sound wave perpendicular to the interface between the layers of the plate construction. The qualitative structure of the considered construction is presented in *Fig. 1*.

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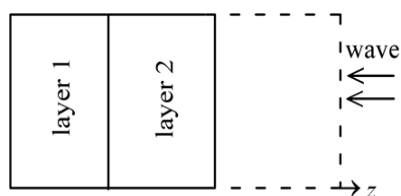


Fig. 1. Structure of the considered construction.

## 2 | Method of Solution

Oscillations in the considered multilayer construction have been determined by solution of the following wave equation.

$$\frac{\partial^2 u(x, y, z, t)}{\partial t^2} = F(x, y, z, t) + \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial^2 u(x, y, z, t)}{\partial x^2} + \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial^2 u(x, y, z, t)}{\partial y^2} + \frac{\partial}{\partial z} \left[ \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial u(x, y, z, t)}{\partial z} \right], \quad (1)$$

where  $E(z)$  is the modulus of elasticity;  $\rho(z)$  is the density of materials of the considered structure;  $\sigma(z)$  is the Poisson ratio,  $u(x, y, z, t)$  is the displacement of the points of the structure during oscillations;  $F(x, y, z, t)$  is the external processing (Knock, sound wave, etc.);  $L_x$ ,  $L_y$  and  $L_z$  are the dimensions of the considered structure in the directions indicated in the indices;  $x$ ,  $y$  and  $z$  are spatial coordinates;  $t$  is the current time. Let us consider the case when the edges of the structure are rigidly fixed and there is no effect on it at the time of the beginning of the considerate processing. Then the boundary and initial conditions for Eq. (1) could be written in the following form

$$\left. \frac{\partial u(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial u(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial u(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad (2)$$

$$u(x, y, z, 0) = 0.$$

Now let us to solve the Eq. (1) using the method of averaging functional corrections [14]. In the framework of the method to obtain the first-order approximation of the desired function  $u(x, y, z, t)$  one shall replace on the not yet known average value  $\alpha_1$  in the right-hand side of the Eq. (1). Then the equation for the first-order approximation of the function  $u(x, y, z, t)$  takes the form

$$\frac{\partial^2 u_1(x, y, z, t)}{\partial t^2} = F(x, y, z, t). \quad (3)$$

Integration of the both sides of Eq. (3) on time leads to the following result

$$u_1(x, y, z, t) = \int_0^t (t - \tau) F(x, y, z, \tau) d\tau. \quad (4)$$

The not yet known average value  $\alpha_1$  of the function  $u(x, y, z, t)$  should be determined using the standard relation

$$\alpha_1 = \frac{1}{L_x L_y L_z \Theta} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} u_1(x, y, z, t) dz dy dz dt, \tag{5}$$

where  $\Theta$  is the continuance of observation of the oscillation in the considered structure. Substitution of the Relation (4) into (5) leads to the following result

$$\alpha_1 = -\frac{1}{2L_x L_y L_z \Theta} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} t^2 F(x, y, z, \tau) dz dy dz dt. \tag{6}$$

The second-order approximation of the function  $u(x, y, z, t)$  was calculated by replacing of the considered function in the right-hand side of the Eq. (1) on the sum of the approximation of the previous order and the average value of the considered approximation  $\alpha_2$ , i.e. by the sum  $\alpha_2 + u_1(x, y, z, t)$ . Then the equation for the second-order approximation of the function  $u(x, y, z, t)$  takes the form

$$\begin{aligned} \frac{\partial^2 u_2(x, y, z, t)}{\partial t^2} = & F(x, y, z, t) + \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial^2 u_1(x, y, z, t)}{\partial x^2} + \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial^2 u_1(x, y, z, t)}{\partial y^2} + \\ & \frac{\partial}{\partial z} \left[ \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial u_1(x, y, z, t)}{\partial z} \right]. \end{aligned} \tag{7}$$

Solution of the Eq. (7) could be written as

$$\begin{aligned} u_2(x, y, z, t) = & \frac{E(z)}{\rho(z)[1-\sigma(z)]} \int_0^t (t-\tau) \frac{\partial^2 u_1(x, y, z, \tau)}{\partial x^2} d\tau + \frac{E(z)}{\rho(z)[1-\sigma(z)]} \int_0^t (t-\tau) \frac{\partial^2 u_1(x, y, z, \tau)}{\partial y^2} d\tau + \\ & \frac{\partial}{\partial z} \int_0^t \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial u_1(x, y, z, \tau)}{\partial z} d\tau + \int_0^t (t-\tau) F(x, y, z, \tau) d\tau. \end{aligned} \tag{8}$$

The average value  $\alpha_2$  of the second-order approximation of the considered function  $u(x, y, z, t)$  should be calculated by using the following standard relation [14]

$$\alpha_n = \frac{1}{L_x L_y L_z \Theta} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [u_n(x, y, z, t) - u_{n-1}(x, y, z, t)] dz dy dz dt, \tag{9}$$

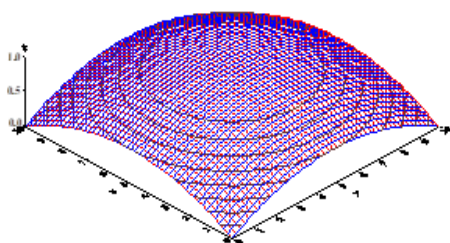
where  $n$  is the order of the required approximation. Substitution of Relations (4) and (8) into (9) leads to the following result

$$\begin{aligned} \alpha_2 = & -\frac{1}{2L_x L_y L_z \Theta} \int_0^{\Theta} t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial^2 F(x, y, z, t)}{\partial x^2} dz dy dz dt - \\ & \frac{1}{2L_x L_y L_z \Theta} \int_0^{\Theta} t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial^2 F(x, y, z, \tau)}{\partial y^2} dz dy dz dt - \\ & \frac{1}{2L_x L_y L_z \Theta} \int_0^{\Theta} t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \frac{\partial}{\partial z} \left[ \frac{E(z)}{\rho(z)[1-\sigma(z)]} \frac{\partial F(x, y, z, \tau)}{\partial z} \right] dz dy dz dt. \end{aligned} \tag{10}$$

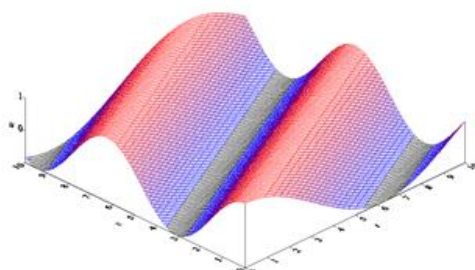
Analysis of the spatio-temporal distribution of the displacement of points of the considered structures during oscillation has been done analytically in the framework of the second-order approximation by the method of averaging the functional corrections. The approximation is usually enough for qualitative analysis and obtaining some quantitative results. The results of the analytical calculations were verified by comparison with results of the direct numerical simulations.

### 3 | Discussion

In this section we analyzed the spatio-temporal distribution of the displacement of the points of the considered multilayer building construction during their oscillations under the influence of a plane wave  $F(x,y,z,t) = A \cdot \exp(i\omega t - k_z z)$ , where  $A$  is the amplitude of the wave,  $k_z$  is the projection of the wave number on the  $Oz$  axis, and  $\omega$  is the wave frequency. *Fig. 1* shows the qualitative spatial distribution of the displacement of points of the considered structure as a function of the coordinates  $x$  and  $y$  at a fixed value of time. *Fig. 2* shows the qualitative spatio-temporal distribution of the displacement of the considered structure points as a function of the coordinate  $z$  and time  $t$  for fixed values of the  $x$  and  $y$  coordinates.

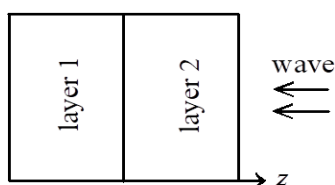


**Fig. 2.** The qualitative spatial distribution of the displacement of the points of the considered structure as a function of the coordinates  $x$  and  $y$  at a fixed time.



**Fig. 3.** The qualitative spatio-temporal distribution of the displacement of points of the considered structure as a function of the coordinate  $z$  and time  $t$  for fixed values of the coordinates  $x$  and  $y$ .

Analysis of the spatio-temporal distribution of the considered displacement shows, that compromise between increasing of sound-insulation of by multilayer constructions and increasing of their complications are using two layer structures, which are presented on *Fig. 4*. These structures consists of load-bearing layer for reflection of received signal (See layer 1 on *Fig. 4*) and facing layer for absorption of the received signal (See layer 2 on *Fig. 4*).



**Fig. 4.** Optimal multilayer structure for sound-insulation conclusion.

In this paper we introduce a model for prognosis of sound-insulation by a multilayer building construction. To analyze the model we introduce an analytical approach for analysis of oscillations of points of the considered structure during sound processing. We formulate recommendations to increase sound-insulation of the considered constructions.

## 4 | Conclusion

This study presented an analytical investigation into the oscillatory behavior of multilayer building structures subjected to acoustic excitation, with the primary objective of improving sound-insulation performance in enclosing and load-bearing constructions. Using the method of averaging functional corrections, a mathematical model was developed to describe the spatio-temporal distribution of displacements within multilayer plates exposed to plane sound waves. The analytical solutions obtained up to the second-order approximation were shown to be sufficient for capturing the essential features of the oscillation process. The results reveal that multilayer configurations play a crucial role in controlling vibration amplitudes and displacement patterns across the structure. In particular, the interaction between layers with different mechanical properties allows part of the acoustic energy to be reflected while the remaining portion is absorbed, leading to improved sound-insulation efficiency. Comparison between analytical predictions and direct numerical simulations confirmed the validity and reliability of the proposed approach for qualitative and quantitative analysis. Furthermore, the study demonstrates that an optimal balance between sound-insulation enhancement and structural complexity can be achieved through the use of two-layer systems consisting of a load-bearing reflective layer and a facing absorptive layer. Such configurations provide effective noise reduction without excessive material usage or design complication. The analytical framework developed in this research can be applied to the preliminary design and optimization of multilayer building constructions, offering practical guidance for engineers and designers aiming to improve acoustic performance in civil and industrial buildings.

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## Data Availability

This study is theoretical in nature and does not involve experimental or publicly archived data. Relevant equations and methods are fully described within the article.

## Authors' Contributions

The author carried out all aspects of the study, including writing the manuscript, and approved the final version for publication.

## Conflict of Interest

The author declares no conflict of interest.

## Consent for Publication

The author has given consent for the publication of this manuscript.

## Ethics Approval and Consent to Participate

No studies involving human participants or animals were performed in this research.

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