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Transverse Vibration Analysis of Saturated Pro-Elastic Beams as a Longitudinal Function Scale

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Abstract

In this research, the free vibrations of beams made of saturated proelastic materials under different boundary conditions have been investigated. The equation governing the transverse movement of the beam is derived based on the Euler-Bernoulli theory of beams and using the relations governing the stress-strain of proelastic materials using Hamilton's principle. Different models of porosity distribution along the longitudinal length of the beam have been considered. The governing equation in different boundary conditions of double-ended, double-ended, and single-ended single-ended boundary conditions is determined by the differential quadratic method for three different types of uniform, linear, and quadratic porosity distribution along the length of the hollow beam. The natural frequencies of free vibration of the longitudinal porous beam have been obtained. The effect of porous material parameters on the change of natural frequencies in different boundary conditions has been concluded. The results obtained have been compared and validated with the results presented by other researchers.

Keywords: Free vibration, Proelastic beam, Euler-bernoulli theory, Natural frequency.

1 | Introduction

The per-elasticity of horns from the continuum theory for the analysis of a pro material includes an elastic matrix that is saturated with fluid [1]. Physically, when a porous material is subjected to stress, the voids undergo volumetric deformation. Since the cavities are filled with fluid, the presence of fluid not only acts as a stiffener of the material but also leads to the diffusion of the fluid inside the cavity between the high-pressure and low-pressure areas [2].

If the fluid is viscous, the behavior of the material system will be time-dependent. Perelastic theories started primarily with problems related to soil and geology. These issues are generally related to massive structures with a three-dimensional nature [3]. Mixing issues related to wave propagation seabed mechanics are some of these issues. The application of the proelastic theory is quite developed today. In the last three decades, the

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proelastic model has been widely and successfully used in biomechanics [4]. There are similarities in mechanical properties, geomechanical behavior and special biological structures. The primary motivation for researching such structures is basically related to the stems and leaves of plants. These elements of plants have two uses, one of which increases the stiffness and strength of the structure, and the other is that it has tissue vessels that bring water from the roots to the leaves [5].

The stems of herbaceous plants and the woody tissue of the stem are filled with water; in early plants, 85% of their weight was water, and later this amount reached about 60%. Such a plant structure is highly non-isotropic. The axial stiffness is more than 20 times larger than the transverse stiffness for wood textures. The main characteristic of such plants is their microstructure, which is designed to transport water axially. Therefore, according to the model of plants, a model of a proelastic beam was modeled where water can move in the axial direction. On the other hand, many researches and articles investigated statically isotropic materials, whose theories are also used for the non-isotropic state [6]. The basic physical assumptions of both are the same, but many of the constants of matter and its mathematical complexities are reduced.

We consider the dimensions of the body in the direction perpendicular to the fluid motion to be close to the isotropic case. In this way, there is no need for new physical assumptions and a number of material parameters are also removed. The constituent materials of the beam elements are considered to be transversely isotropic in the transverse section of the plate. This research provides a method and theoretical principle for researching the mechanical behavior of parts made of such materials [7]. These are not limited to plant stem and trunk materials, which was the main motivation for the model of such materials, but artificial materials with similar behavior are also very important. Peru materials have different behavior than non-Peru materials [8].

2 | Per-elasticity

Proelastic materials can be divided into two general categories: natural and artificial. Natural materials such as wood, stone, and soil are naturally formed around us, and by conducting tests on them, their characteristics, including porosity, are measured. However, the synthetic handle is determined and made according to the needs of their specifications [9].

3 | Classification of Natural Pro-elastic Materials

Porosity is classified on different basis such as the size of the holes and the time of their formation. In the classification based on the size of the holes, the porosity is divided into two types: macroscopic and microscopic. In the division based on the time of formation of pores in rock or sediment, porosity is divided into primary and secondary types [10].

4 | Per-elasticity Types Based on Formation Time

Also, per-elasticity can be classified into primary and secondary categories based on the time of formation, where primary porosity is created at the same time as sedimentation (sandstone porosity is generally primary). Secondary per-elasticity is formed after sedimentation and burial of sediments (porosities of carbonate rocks are mainly of secondary type).

Analysis of the behavior of porous materials

Porousness is a continuum theory for the analysis of a porous material consisting of an elastic matrix with pores saturated with fluid. Physically, the behavior of the porous structure is such that when the material is subjected to stress, the deformation of the matrix causes a change in the volume of the pores. Since the pores are filled with fluid, they not only stiffen the body but also allow the fluid to move from the high-pressure area to the low-pressure area. If the fluid is viscous, the behavior of the material is time-dependent. The free movement of the fluid in the porous body improves its mechanical properties. There are two main mechanisms in the reaction between the fluid inside the porous body and the solid porous body [11].

I. Increasing the pressure of the porous body causes the pores to expand.

II. Squeezing the holes causes the fluid to increase in pressure.

This coupling mechanism gives the porous material a distinct time-dependent property. If the fluid pressure is increased in increments, it causes the fluid to force its way through any pores, which leads to the loss of the porous material over time. It seems that porous material is much more pliable in the drained state, i.e., the state in which the fluid can escape from the rock, than in the undrained state, i.e., the state in which the fluid cannot escape from the rock. The first theory of the effect of fluid on the quasi-static deformation of soil was made by Terzaghi in 1923, when he presented the one-way consolidation model. This theory was generalized to three directions by Crandoli in 1936. Bayot, an oil exploration engineer for Shell, first developed the theory of linear elasticity in 1935 and 1941, which includes the two properties mentioned above. In addition, many other theories have been proposed in this field, but none of them is as practical as Biot's theory. The basic phenomenological model for such materials was proposed by Biot [12]. His research has been of interest in geological discussions over the years in the fields of soil consolidation and wave propagation.

5 | Effective Factors in Primary Porosity

Grain size

As the grain size decreases, the amount of porosity increases but the permeability decreases. By increasing the size of the grains, the amount of useful porosity increases and the permeability increases. As shown in *Fig. 1*, in fine-grained sediments, the channels connecting the holes are very small, and the high capillary pressure in the walls of these channels prevents the passage of liquids.

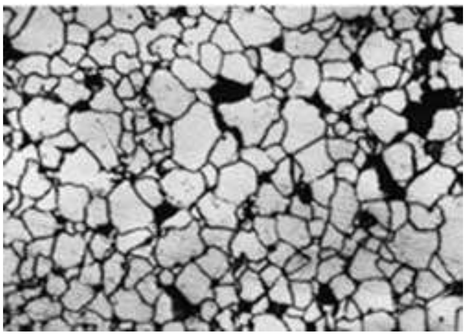

Fig. 1. The number of seeds is much more than the number of fluid penetration channels in the holes.

Adaptation

As shown in *Fig. 2,* The better the fusion, the higher the porosity and permeability because in the rocks with bad fusion, fine grain particles or the matrix between the coarse grains fill and reduce the porosity and permeability.

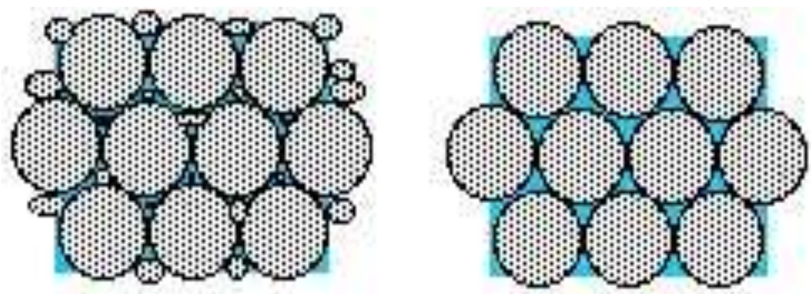

Fig. 2. The presence of particles and blockage of fluid channels.

The shape of the seeds

If the grains have good roundness and sphericity, their arrangement is in such a way that they are closer together (dense arrangement) and reduce the amount of porosity and permeability. As shown in *Fig. 3*, slightly angular grains are better for initial porosity.

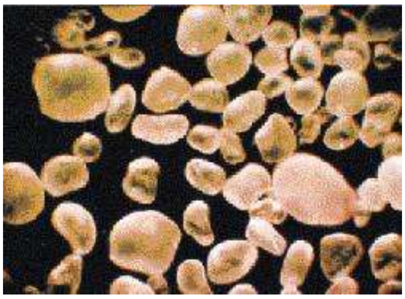

Fig. 3. The shape of the seeds that make up the Peru substance.

Secondary porosity is more important than primary porosity. The most important factors of secondary porosity are fracture, dissolution and replacement of chemical substances.

Fracture

It includes the cracks in the stone. Although fracture porosity is usually low (about 1 to 2 percent) However, fractures play an important role in the passage of fluid through the rock and dramatically increase the flow capacity within the rock.

Liquidation

It is a chemical reaction in which water and carbon dioxide dissolved in it react with calcium carbonate and form calcium bicarbonate which is soluble. This reaction increases the porosity of limestone.

Chemical substitution

It is a chemical reaction in which another ion replaces one ion. As a result of this replacement, we will have a reduction in the size of the new composition.

6 | Classification of Synthetic Per-elasticity Materials

Artificial synthetic materials are new and widely used materials today, which are produced and used in two structures, porous materials in nano size and porous materials in macroscopic dimensions [11].

7 | Foams and Their Use

Foam materials are among new and advanced materials. They will have a lot of potential for development in the coming years. The family of foam materials is divided into three categories:

I. Metallic foams.

II. Ceramic foams.

III. Polymer of foams.

7.1 | Metal foams

Metal foams have various applications in industries and these applications are increasing in proportion to the increase and innovation of new production methods of these materials.

Applications of metal foams include:

Land and rail transportation

What is important in the land transportation system is air pollution and safety issues. The use of aluminum foams has made it easier to manufacture the bodies of cars and trains, and this has effectively reduced the cost of production. On the other hand, the weight of these devices has also been reduced, resulting in reduced fuel and energy consumption, and the issue of environmental pollution is also partially resolved. Aluminum foams used in the parts and bodies of cars and trains are 10 times stronger and have greater strength than steel parts, and on the other hand, they are 50 percent lighter. Today, 20% of the car body can be made of aluminum foam, which reduces the weight of the car by 60 kilograms and, as a result, reduces fuel consumption, so it can be said that about 2.6 miles more can be traveled per gallon of gasoline. Another significant issue in the ground transportation system is safety. Due to their ability to absorb mechanical energy in all directions, metal foams are easily deformed and compressed during accidents and collisions with obstacles. On the other hand, since the elastic movement of these foams is very limited and low, during impact, the foam is prevented from moving and returning to its original position, as well as from accelerating the vehicle in other directions (the opposite direction of the impact), and from whiplash-like motion. Also, metal foams do not noticeably and significantly lose their properties due to severe and rapid impacts and, as a result, are more resistant than other materials.

Another advantage of metal foams is their sound insulation. By using these foams, which are used in the construction of the body, engine, hood and intermediate parts between the engine and the room, noise pollution caused by the engine can be prevented. As will be mentioned, these foams absorb the sound of the car engine due to their structure of holes and prevent it from entering the room and the seating area of the passengers.

Marine transportation

Aluminum foams have been used for the construction of various parts of ships and other marine vehicles due to their very large weight reduction and, on the other hand, increased corrosion resistance, as well as weldability and formability. Sandwich structures save more than fifty percent in weight and volume of the structure compared to other metal structures. The use of metal foams reduces noise pollution by 20 percent. In addition, the strength, fire resistance and corrosion resistance increase. The unique characteristics of sandwich structures have led the US Navy to start its study of building steel sandwich structures with foam cores and laser welded since 1987 and use them to build various parts of the ship.

Another use of metal foams in the marine industry is the construction of ship walls and hulls, especially wave platforms. When large, powerful waves enter the porous structure of these foams, their speed and intensity are reduced by colliding with the cell walls and creating friction, preventing them from damaging ships and even piers.

Air transport

In the air transport sector, as in the sea and land transport sectors, weight reduction is very important. So much so that it can be said that this is one of the main goals of aircraft designers; by using aluminum foams, in addition to achieving the necessary strength, weight reduction and, as a result, fuel consumption reduction are also achieved. By reducing fuel consumption, on the one hand, the cost of using the aircraft is reduced, and its attractiveness is increased so that more passengers, goods and postal items refer to it for transportation, and on the other hand, the amount of environmental pollution is reduced [11].

Today, the use of aluminum foam sheets or foam boards instead of honeycomb structures in the construction of the aircraft body has reduced the cost of building the body, and, on the other hand, the strength of the structure, especially its creep resistance, has increased sharply.

Use in the construction industry

Metal foams are often used to make sound insulation in buildings. In addition to being strong, the use of these foams also makes the building lighter. Metal foams are also fire-resistant and do not produce dangerous toxic gases.

Use as sandwich structures

A sandwich panel is a multi-material structure or structure. This structure consists of two sheets that surround a central core and are so-called sandwiched.

The most important features of sandwich panels that have attracted great attention include:

I. Low weight, strength, high stiffness and hardness, thermal properties suitable for thermal insulation, sound wave absorption, electromagnetic wave absorption, energy absorption, etc.

II. Sandwich structures filled with metal foam cores are widely used in the construction of automobile bodies, rail vehicles, and ships.

Aerospace industry

The use of lightweight metal foams in the aerospace industry is similar to the automotive sector. In aircraft, replacing expensive honeycomb structures with foamed aluminum sheets or foamed boards can lead to a favorable price reduction. On the one hand, creep resistance is improved, and on the other hand, the important advantage of these foams is the isotropy of their mechanical properties.

In space applications, the use of highly reactive but very lightweight foams such as aluminum has been considered. These alloys are usually not usable due to their high reactivity, but they can be useful in vacuum conditions.

Biomedical industry

Titanium or cobalt-chromium alloys are used in the manufacture of artificial teeth or prosthetic limbs due to their biocompatibility. Usually, porous titanium or titanium foam is the material that can be used for such applications. There is no consensus on how prosthetic limbs should be designed to maximize the stability and function of the intended limb.

For example, according to one opinion, the modulus of artificial teeth should match the modulus of the jaw bone. Also, knowing the relationship between the modulus and density of metal foams is one way to make it easier to fabricate prosthetics with consistent moduli, ensuring biocompatibility and stimulating bone growth within the open porosity.

Filters and separators

There are two types of filters: filters that retain and separate solid particles or sediments in a liquid and filters that retain and separate solid or liquid particles in a gas (smoke or mist).

Examples of the first type are filters used to clean polymer melts, to separate yeast from beer, or to refine oil. The second type of filter includes filtering diesel fumes or dewatering in airlines. The most important properties of foam filters are good particle separation, good particle retention, cleanability, mechanical properties, corrosion resistance, recyclability, and reasonable and economical production cost.

Ceramic foams

Ceramics is derived from the Greek word keramos, which means pottery or baked objects. In fact, the origin of this science is the pottery made by early humans. In fact, before the discovery and use of metals, humans used clays due to their abundance and abundance, as well as their excellent formation when mixed with water and their relatively low baking temperature. Aluminosilicates, which are considered clay soils themselves, are made of the elements aluminum, silicon, and oxygen, which together make up about 85 percent of the Earth's solid crust. These three elements are the most abundant elements in the Earth's crust.

Engineered ceramic foams can be divided into two categories: open and closed porosity. The pores and solid phase of open-pore foams are interconnected in three directions, and the pores lead to the outside of the material. In closed-pore foams, they are separated from the outer surface of the material due to the presence of solid layers. This group of ceramic materials has received more attention due to its suitable properties such as thermal stability, low thermal conductivity, low density, high specific strength, resistance to chemical attacks and high permeability. The aforementioned properties have led to this group of materials being used in many industrial applications, such as filtering molten metal and hot gases, catalyst supports, fuel cell electrodes, battery separators, chemical sensors, thermal insulation, biomaterials for bone replacement and pharmaceutical industries. This article reviews the methods of manufacturing ceramic foams and discusses some of their key properties and applications.

Applications of ceramic foams

Today, ceramics are widely used in human daily life. Various types of containers, sanitary ware, and construction materials are among the daily uses of ceramics. In nuclear power plants, thick ceramic parts are used as reactor shields to prevent the emission of radioactive radiation outside the reactor, and in the electronics industry, they are used to make capacitors.

Polymer foams

In the case of polymer foams, the solid phase is made of polymer. In a foam mass, there can be two types of empty spaces in the polymer part, which are called cells. Hence, there are two types of cells in foams, including open and closed. In the case of open-cell foams, the gas phase is also continuous, while in closed-cell foams, the gas phase is discontinuous. The type of cell greatly changes the mechanical and thermal properties of polymer foams.

Types of polymer foams

Polystyrene foam, polyurethane foam, phenolic foam, urea formaldehyde foam, polyvinyl chloride vinyl alcohol-formaldehyde foam, epoxy foam, and other foams. Among these foams, items one to five have been used in building insulation and sandwich panels. It is worth noting that in sandwich panels, open-cell rigid foams are often used. The polymers used in the manufacture of these foams are divided into two general categories: thermoplastic and thermosetting. Polystyrene and PVC foams are examples of the first (thermoplastic), and phenolic, urea-formaldehyde, and polyurethane foams are examples of the second (thermoplastic). Therefore, depending on the type of polymer used in the foam, its production method varies. What is important about different foams is the type of polymer and the type of gas in its cells. These two factors determine the thermal conductivity or the ability of a foam to act as a thermal insulator.

Polystyrene foam

Expanded polystyrene, or EPS for short, is fire-resistant polystyrene, a light, white granule. This material was first produced in 1950. The expansion of this product is due to the presence of a certain amount of pentane gas, which is trapped inside it in an undissolved state during production. This gas is released from the polystyrene beads by the heat of water vapor and causes them to expand. As a result of this gas release, the volume of the polystyrene beads increases by up to 40 times their original size. After the expansion operation, the expanded beads are molded according to the type of application.

Expanded polystyrene is a product that does not cause problems for the environment and humans. The lack of harm to the environment is evident in all stages of manufacturing, use, and recycling or disposal of this product. Fire-resistant polystyrene foam is actually fire-resistant polystyrene.

In the early 1950s, BASF developed a two-stage process for producing polystyrene foam. In this process, the first stage involved preparing beads containing a uniform distribution of the blowing agent by the suspension polymerization method of styrene monomer, which in the second stage was processed inside a mold. The ease of producing products in any shape and size is one of the advantages of this method that led to its development. With the introduction of polystyrene foam to the market and its applications in the construction industry and other industries, this foam has found its place in this industry.

Features of fire-resistant polystyrene foam:

I. Fire-resistant.

II. Good thermal insulation.

Usages and applications

Expanded polystyrene has been used in many packaging applications for more than 50 years. In addition, this product is used in the manufacture of thermal insulation for buildings, cold stores, freezing tunnels and containers for transporting materials. Most manufacturers in various industries need precise packaging to deliver their products to prevent them from being damaged during transportation and delivery. Features such as low weight, dimensional and thermal resistance, moisture stability, good impact resistance and excellent moldability of expanded polystyrene have played a very important role in the packaging and protection of sensitive electronic components. In addition, the resistance to moisture and the non-degradability of expanded polystyrene have made this product an excellent product for packaging pharmaceutical and food products.

Another application of this product is its use in insulation. Its good thermal resistance and its non-degradability over time have led to the use of this material in the insulation of cold stores. Of course, in recent years, polystyrene blocks have replaced clay in the walls of houses and buildings, which has reduced the cost of workforce, reduced the consumption of other equipment, saved the volume and cost of concrete, saved the cost of the skeleton and foundation, and saved the overall cost of roof construction.

Compared to ceramic and polymer foams, metal foams are newer and their properties and applications are being developed, so it seems that a bright future is waiting for these materials. Metal foams have been noticed since 1950; the passage of time and more familiarity of scientists provided the basis for the development of these materials; since the end of 1980, extensive research has been done in the field of metal foams, and among the production methods, metal foams produced in Melting method was used as the main commercial product in the field of metal foams.

Metal foams have unique properties, including:

I. Excellent mechanical properties.

II. Suitable thermal properties.

III. Sound properties.

IV. Excellent corrosion resistance.

These materials are of great interest in the transportation industry due to their high strength and lightweight, and they are used as sound insulation due to their ability to absorb high energy. Metal foams are used in the manufacture of car bumpers due to their energy absorption, which, in addition to the mentioned applications, are used in aerospace and other industries.

Nanoporous materials

Nanopores or nanoporous materials are pores smaller than 100 nanometers and have special applications in filtration at the atomic and molecular levels. These materials have interwoven pores and their pores have a repeating pattern of transition in three-dimensional space and do not require any special order or rule in the arrangement of pores to be called nanoporous.

Nanoporous solids can have a variety of compositions, such as carbon, silicon, silicates, polymers, ceramics, metals and organometallic compounds. Many porous materials are thermodynamically unstable and will collapse as soon as the kinetic energy of the boundaries is overcome.

For example, porous silicates are semi-stable materials that, as the temperature increases and the melting point is reached, the initial particles in the network melt. At this stage, metal separation occurs, and a nanoporous silicate phase is formed. Controlling the interface energy and the quasi-stable state of nanoscale pores is of particular importance when creating nanoporous materials.

Nanoporous materials can be classified according to two main criteria:

Classification by pore size

I. Materials with fine pores, with pore sizes of 0-2 nm.

II. Materials with medium pores, with pore sizes of 2-40 nm.

III. Materials with coarse pores, with pore sizes larger than 50 nm.

Classification by material network

One of the most important goals in the field of nanoporous materials is to achieve a chemical composition with a set of pores in the structure

and this causes the materials to be divided into two categories: inorganic and organic (such as polymers).

Properties and applications

The main application of nanopores is to lighten minerals in a way that maintains and increases the stability of these chemical compounds. For example, aerogels are extremely light nanopores and can easily support up to 100 times their own weight.

The surface area of a solid increases after becoming nanoporous, which improves its catalytic, adsorption, and adsorption properties. The surface area of nanoporous solids is often on the order of a few hundred square meters per gram. An example of such materials is zeolites. Zeolites are minerals with nanoscale pores and have been used as catalysts for decades. The adsorption and surface adsorption properties of these materials indicate their ability to solve environmental problems (such as removing heavy metals such as mercury and arsenic). Due to the high surface adsorption of these materials, they can also be used as molecular sieves. In this case, these materials separate some molecules through surface reactions. Due to their high free surface area, these materials can also play an important role in catalytic reactions.

8 | Conclusion

The use of new materials has become inevitable and undeniable with the increasing progress of science. Today, metals such as iron and copper can no longer meet human needs alone; as a result, solving these needs requires more focus on research and investigation in the field of new materials. The theory of perelasticity is a broad theory about the interactions between the liquid and solid phases of a porous medium saturated with liquid. It also models the interactions related to deformation and liquid flow in porous materials saturated with liquid. Changing the shape of the medium affects the liquid flow and vice versa.

References

[1] D'angelo, J., Case, E. D., Matchanov, N., Wu, C. I., Hogan, T. P., Barnard, J., … & Kanatzidis, M. G. (2011). Electrical, thermal, and mechanical characterization of novel segmented-leg thermoelectric modules. *Journal of electronic materials*, 40(10), 2051–2062. https://doi.org/10.1007/s11664-011-1717-7

[2] Givi, A. N., Rashid, S. A., Aziz, F. N. A., & Salleh, M. A. M. (2010). Assessment of the effects of rice husk ash particle size on strength, water permeability and workability of binary blended concrete. *Construction and building materials*, 24(11), 2145–2150. https://doi.org/10.1016/j.conbuildmat.2010.04.045

[3] Zhao, J., Jiang, Y., & Li, X. (2023). Flexural behavior of concrete beams reinforced with high-strength steel bars after exposure to elevated temperatures. *Construction and building materials*, *382*, 131317. https://doi.org/10.1016/j.conbuildmat.2023.131317

[4] Singh, A., Kumar, R., Mehta, P. K., & Tripathi, D. (2020). Effect of nitric acid on rice husk ash steel fiber reinforced concrete. *Materials today: proceedings*, *27*, 995–1000. https://doi.org/10.1016/j.matpr.2020.01.310

[5] Srinivasamurthy, L., Nataraja, M. C., & Srinivasan, K. (2024). Investigation on flexural behavior of conventionally reinforced, steel fiber-reinforced, and post-tensioned geopolymer concrete beams. *Journal of the institution of engineers (India): series a*, *105*(1), 129–150. https://doi.org/10.1007/s40030-023-00772-3

[6] Sharma, R. K. (2014). Effect of substitution of cement with rice husk ash on compressive strength of concrete using plastic fibres and super plasticizer. *KSCE journal of civil engineering*, *18*, 2138–2142. http://dx.doi.org/10.1007/s12205-014-0634-8

[7] Heidary, F., & Reza Eslami, M. (2006). Piezo-control of forced vibrations of a thermoelastic composite plate. *Composite structures*, *74*(1), 99–105. https://doi.org/10.1016/j.compstruct.2005.03.011

[8] Zareei, S. A., Ameri, F., Dorostkar, F., & Ahmadi, M. (2017). Rice husk ash as a partial replacement of cement in high strength concrete containing micro silica: evaluating durability and mechanical properties. *Case studies in construction materials*, *7*, 73–81. https://doi.org/10.1016/j.cscm.2017.05.001

[9] Li, S., Su, H., & Cheng, C. (2009). Free vibration of functionally graded material beams with surface-bonded piezoelectric layers in thermal environment. *Applied mathematics and mechanics*, *30*(8), 969–982. https://doi.org/10.1007/s10483-009-0803-7

[10] Quan, J. R., & Chang, C. T. (1989). New insights in solving distributed system equations by the quadrature method—I. Analysis. *Computers & chemical engineering*, *13*(7), 779–788. https://doi.org/10.1016/0098-1354(89)85051-3

[11] Shu, C., & Du, H. (1997). Implementation of clamped and simply supported boundary conditions in the GDQ free vibration analysis of beams and plates. *International journal of solids and structures*, *34*(7), 819–835. https://doi.org/10.1016/S0020-7683(96)00057-1

[12] Biot, M. A. (1941). A mechanical analyzer for the prediction of earthquake stresses. *Bulletin of the seismological society of america*, 31(2), 151–171. https://doi.org/10.1785/BSSA0310020151

[13] Krzysztof Magnucki, P. S. (2004). Elastic buckling of a porous beam. *Journal of theoretical and applied mechanics*, *42*(4), 859–868. https://b2n.ir/g99346

[14] Wang, X.; Bert, C. W. (1993). A new approach in applying differential quadrature to static and free vibrational analyses of beams and plates. *Journal of sound and vibration*, *162*(3), 566–572. https://doi.org/10.1006/jsvi.1993.1143